A NOTE ON BIB DESIGNS

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Das has stated in Lemma 3 of [1] that $k(v-1)/r$ must be an integer excepting those designs which can be obtained by repeating a design. Kageyama and Ishii, [3] Proved the falsity of the lemma by considering $v=22$, $b=44$, $r=14$, $k=7$, $\lambda=4$ which can not be obtained by repeating the symmetric design $v=b=22$, $r=k=7$, $\lambda=2$ because the symmetric design does not exist. With this example they reframed the original lemma as follows. "In all BIB designs with parameters v, b, r, k, λ excepting those with $v=v$, $b=c\,b_1$, $r=cr_1$, $k=k$ and $\lambda=c\lambda_1$ for some integer c, $k(v-1)/r$ is an integer".

The first object of this paper is to show that not one but a series of examples prove the falsity of Das's lemma. Consider the series due to Sprott [4] which can be constructed if $2(k-1)=p^n$ where p is prime.

$$v=2k;\ b=4(2k-1);\ r=2\,(2k-1);\ k;\ \lambda=2\,(k-1)$$

In all such designs for odd values of k, $k(v-1)/r$ is not integer. Infact Kageyama and Ishii [3] reframed lemma is also not totally correct because for many designs whose parameters are of the type $v=v$, $b=cb_1$, $r=cr_1$, $k=k$, $\lambda=c\lambda_1$, $k(v-1)/r$ is an integer. Consider the design $v=16$, $b=40$, $r=15$, $k=6$, $\lambda=5$ which is of the above form where $c=5$, but $k(v-1)/r$ is an integer. Similarly (121, 132, 60, 55, 27) is of the above form with $c=3$ but $k(v-1)/r$ is an integer. These observation make us to reframe the lemma as follows. "In all BIB designs with v, b, r, k, λ excepting those with $v=v$, $b=cv$, $r=ck$, k, λ, c and v are even integers $k(v-1)/r$ is an integer".

Das [1] in section 3(b) has stated that λ is a factor of $(k-1)$ if r and k have no common factor. Kageyama and Ishii [3] have taken an example of an unknown design to show that λ is not a factor of $(k-1)$ even though r and k have no common factor. In fact, the above series given by Sprott [4] shows that λ is not a factor of $(k-1)$ even though r and k have no common factor.

In section 3(*a*) Das has proved that when r and k have no common factor $b-r \geqslant (v-1)$ and in section 5(*a*) he proved the conditions for resolvability. Third object of this note is to combine both the results and give simpler proofs.

Following Das [1]

$$\frac{b-r}{v-1} = \frac{r-\lambda}{k} = \frac{r^2-b\lambda}{r} = \frac{n_1}{s}$$

where n_1 is the H.C.F of $r-\lambda$ and $r^2-b\lambda$ and s that of r and k. When $s=1$, $r-\lambda \geqslant k$.

Stanton [5] proved that this inequality is equivalent to $b-r \geqslant v-1$. When r and k are prime to each other, we have b is divisible by r and thus satisfies the conditions.

Das and Kulshresta [2] considered the series v, pv, pk, k, λ and constructed initial blocks for some BIBDS. In a lemma, there they showed that $(v-1)/p$ is an even integer. Luckily for them the designs given in Table 1, all v satisfy that relation. The method fails when v and p are both even.

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